

AN INTERFERENTIAL INVESTIGATING RELATION
INTERFERENCE METHOD FOR STUDYING THE CONNECTION BETWEEN
THE STRUCTURE OF WAVE SURFACES AND THE
STRUCTURE OF ENERGY-DISPERSION SPOTS

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(ZhTF XVIII-3, Mar'48)

1. Theory of the Method

Let a certain plane intersect the conjugation of the normal to a juncture of the wave surfaces (rays) which have an arbitrary structure; and let us consider the point, or ~~the~~ the spot, of the energy dispersion, which is represented in this plane as a monochromatic point emitter. The structure of the energy dispersion points depends on the structure of the wave surface and on its limits. Within the limits of the region in which the plane cuts the macrostructure and a micro-junction of the normal to the wave surfaces, the macro-structure of the energy-dispersion points exist. The macrostructure is determined by the location of each point of intersection of the plane with the normal to the relative wave surface, relative to the section of the normal containing centers of curvature of various normal cross-sections which corresponds to the element of the wave surface, and relative to the length of this section. The macrostructure basically forms the energy-dispersion points and particularly determines the location and characteristics of its brightest elements. Imposed on the macrostructure, the periodic microstructure appears as the result of an interference of disturbances, occurring in every examined point of the dispersion spot, under study, by the Huygens-Fresnel principle, from all wave surfaces. Calculation, with Guignen-Frenell's principle, of the distribution of energy in the dispersion spots for a point monochromatic emitter can and cannot give a picture of the macro-structure of the energy-dispersion spots found in testing since, in an observed in tests, energy maxima, the case of a point emitter, maximum energies in corresponding places

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of the spots can not generally speaking be separated according to maximum brightness as compared with other diffraction maxima. However, by ~~way~~ passus with the increase in the degree of the disc diameter growth representing the emitter, the dispersion-spots' diffraction contours of the dispersion spots coinciding with its macro-structure picture, according to the dimensions of the diameter of and the disc must remain sharply outlined during the period that other diffraction contours of the dispersion spots are fading. An examination of the geometry of wave surfaces permits a localization of the picture of the macrostructure of the energy-dispersion spots and to describe its characteristics.

A.

CLASSIFICATION OF THE ELEMENTS ~~OF~~ MACROSTRUCTURE OF ~~AN~~ ENERGY-DISPERSION SPOTS

There exist three specific types of energy concentration in the elements of dispersion points which form their macrostructure. These are: the focal nucleus, elements of the picture of the focal line, and diffusion elements. The elements of dispersion spots belong to the given type of concentration, have characteristic properties which determine their role in the formation of the picture.

It is convenient to examine wave surfaces which are made up of elements. The normal to each of elements coincides with the ray emanating from the corresponding point of the 'discharge focus' of the aberration system. Let us consider any arbitrary element of a wave surface and the normal be erected on it. The curvature radii of various normal cross-sections of this element can be different (astigmatic element) or can be alike (anastigmatic element).

The curvature centers of various normal cross-sections of the element of the wave-surface form normal segments, ^{at the} limited by the centers of curvature pairs of its main cross-sections by curvature centers (astigmatic difference).

(in the segment of) pair of
 Each point of the curvature centers segments correspond to a normal cross-section of the wave-surface element whose curvature centers are found located

in this point. The angle between these normal cross-sections changes varies during transition from point to point of the curvature-center segment containing the center of curvature at the passage from point to point of the curvature-center segment containing the main cross-section of the wave-surface element, and consequently in the case of the astigmatic wave-surface element, becomes zero twice.

Wave-surface elements, in which the system converts the specific incident upon wave falling on it, can be distinguished thus:

- 1) In the sense of the length of the normal segment having curvature centers of the normal cross-section of the wave surface element. The wave surface can in particular have elements for which this length becomes zero (anastigmatic elements of wave-surface);
- 2) in the shape of the location of this segment of the centers of curvature center on the normal;
- 3) In the sense of the orientation of pairs of main mutually-orthogonal cross-sections of the wave-surface element.

The geometric place of the curvature centers of the main cross-sections of the wave-surface elements forms the caustic (focal) surface.

Each astigmatic wave-surface element forms two points of this surface, shifted (points) relative to another along the corresponding normal.

Each anastigmatic wave-surface element forms one point of the caustic surface -- its point.

The wave surface can contain anastigmatic elements having a common center of curvature-center which forms, consequently, a general point of the caustic surface. Curvature centers of various anastigmatic wave-surface elements do not have to coincide. In this case they form

various points of the caustic surface which can appear scattered in space on the corresponding normals in any way, and in particular, in one focusing plane.

The intersection of each of the normals with focusing plane deter-

mimes a certain point of the dispersion spot. Each point of the dispersion spot can appear variously distributed along the normal, to which it belongs, relative to the segment of the curvature centers of the corresponding wave-surface element. The length of this segment can vary for various wave-surface elements. By using these characteristics, one can classify the elements of the energy-dispersion spots can be classified.

1. Focal Nucleus. This is an element of the dispersion spots around its point coinciding with curvature center of the anastigmatic wave-surface element. The dispersion spot can contain several focal nuclei, one focal nucleus, or none.

2. Element of the focal line. This is an element of the dispersion spot around its point coinciding with curvature center of one of the pairs of main cross-sections of an astigmatic wave-surface element. The picture of focal lines of the energy-dispersion spots is formed in the intersection of the plane with the caustic surface. Each point of this picture is connected by a ray with a certain point of the discharge pupil of the system.

The focal-line element can be degenerate at the point where the caustic-surface cavity degenerates into the line formed in the intersection with the plane at the referred-to point. For example, in the case of spherical aberration, one of the caustic-surface cavities degenerates in a straight line coinciding with the axis of the system.

3. The diffusion element of the dispersion spot is formed around its point, which does not belong to the caustic surface. This is the internal diffusion element, located when the point is found within the segment of the curvature centers of the corresponding wave-surface element. In the opposite case it is the external diffusion element. As a result of restricting the wave surface, the energy can be present in the points which do not relate to the normals to the wave surfaces. The dispersion-spot element around such points are diffusion

diffraction
defractionary elements.

Normals to various wave-surface elements can be intersected and, consequently, can be superimposed on diffusion-spot elements of various or similar types.

If the normals to the anastigmatic wave-surface element are moved, then the corresponding dispersion-spot element in various planes gradually becomes an external diffusion -- focal nucleus -- external diffusion element. In the case of the astigmatic wave-surface element, the dispersion-spot element gradually becomes ^{successively} an external diffusion -- focal-line element -- internal diffusion -- focal-line element -- external diffusion element.

The energy-dispersions spot can contain elements of all three types of concentration or only certain of them, which correspond to

~~SPACE~~ the wave-surface structure.

(B) Wave Surfaces Aberration, Isophase lines on the Pupil, and classification of the discharge pupil

(CAPS)

~~SPACE~~ Let us study the properties of the discharge pupil of an optical system in the vicinity of its point (O) from which a ray enters a given point (P) of the dispersion spot.

Let a specific light wave, whose center coincides with the focus, encounter an optical system and be converted by it.

We can compare the actual wave surface, emanating from the system, with spheres whose centers coincide with various points (P) of the dispersion spot in the given focus plane. The physical sense of such a comparison is included in the fact that the ^{real} wave surface is compared with wave surfaces which exist in the ideal system whose centers of curvature coincide with the studied points (P) of the dispersion spot. In the general case ^{of an} the aberration of the system there are no ^s based (apriori) giving preference to a comparison with some kind of sphere of comparison.

About any point (Ω) of the dispersion spot, there can be constructed, as around the center of the sphere of a certain radius r , a comparison sphere tangent to the wave surface at point (0) whose normal contains this point (Ω). Deviations of the wave surface from the comparison sphere are read along the radius of the latter and is a value of the wave aberration I_{Ω} of the system for the point (Ω). The intersection of each radius of comparison sphere with discharge pupil forms its point for which the wave aberration is determined along this radius. Thus, there can be constructed, in functions of the coordinates of the points of the 'discharge-pupil' (or some other plane) of the system, a surface of wave aberration I_{Ω} , for the studied point (Ω) of the energy-dispersion spot, which characterizes the deviations of the system from the specific wave surface whose center coincides with the studied point (Ω). The surface of the wave aberration corresponds to a set of isophase (isoc aberrated) lines on the discharge pupil, of the system for the point (Ω). Each such line ^{is} the pupil point with a uniform value of the wave aberration for the point (Ω). Let the element (0) of the wave surface radiate from the arbitrary element (0) of the discharge pupil system. The point (Ω) of the intersection of the normal to the element (0) of the wave surface with a certain plane represents the studied point of the energy-dispersion spot in this plane.

Let the center of the comparison sphere, tangent to the wave surface at the point (0), be located in this point (Ω). The comparison sphere assigns for the point (Ω) a surface of wave aberration I_{Ω} and a set of isophase lines on the pupil. Element (0) of the surface of wave aberration corresponds to the element (0) of the wave surface.

Many comparison spheres can be constructed which are tangent to the wave surface at the point (0) and whose curvature centers are disposed at various points (Ω) of the examined normal under consideration.

What are the properties of the element (0) of the surface of wave-

surface aberration and isophase lines in the vicinity of an arbitrary point (0) of the discharge pupil, which point coincides with the center of the compared sphere due to the properties of the element (0) of the wave surface and with the position of the point (Ω) on the normal?

Consider Let us examine the plane cross-section of the pupil element (0), (section) path of which is represented as the intersection with the pupil plane of a certain normal cross-section of the element (0) of the wave surface. Let the studied point (Ω), coincident with the center of the compared sphere, be shifted along the normal by a magnitude Δ to a distance from the curvature center of the examined normal cross-section of the wave surface element (0). det.

With a geometric construction it is easy to see that, in the vicinity of the point (0) of the pupil plane (or any other plane), the wave aberration $I_{\Omega} = I_{\Omega}(\sigma)$ in the examined cross-section (0) of the pupil is described by the following expression:

$$I_{\Omega}(\sigma) \Big|_{\sigma=0} = \frac{\Delta}{2r^2} \sigma^2 \cos^2 \gamma \quad (1)$$

P 306
(ZHTF)
18-3

where σ is the linear distances in the examined cross-section of the pupil element (0); r is the radius of the compared sphere whose curvature center coincides with the point (Ω) and which is tangent to the wave surface at the point (0); γ is the angle between the straight lines which form the plane of the normal cross-section in the plane which is tangent to the wave surface at the point (0) and in the pupil plane; Δ is the radius of curvature of the examined cross-section of the wave-surface element (0). It follows from formula (1) that when $\sigma=0$, that is at the point (0) of the pupil, in any cross-section (i.e. with any value of Δ)

we have the following relations:

$$I'_{\Omega}(\sigma) \Big|_{\sigma=0} = 0 \quad \text{and} \quad I''_{\Omega}(\sigma) \Big|_{\sigma=0} = \frac{\Delta}{r^2} \cos^2 \gamma \quad (2)$$

formulas.

It is clear from (2) that element (O) of the surface of the wave aberration is always ~~extreme~~^{an inflection} for any point (σ) belonging to the normal to wave surface at point (O). The sign of this extreme in any cross-section of the element (O) of the pupil is determined by the sign of Δ for the corresponding normal of the cross-section of the wave-surface element (O). In cross-sections of the pupil element (O) corresponding to the normal cross-sections of the wave-surface element (O) whose (cross-sections) curvature ~~centers~~ coincide with the point (σ_0), (i.e. for which $\Delta = 0$) there is ~~an inflection~~^{an inflection} point; here simultaneously the relations:

$$\left. l'(\sigma) \right|_{\sigma=0} = \left. l''(\sigma) \right|_{\sigma=0} = 0. \quad (2') \quad p307$$

Let us examine the properties of the isophase lines in the vicinity of the point (O) in the plane tangent to the wave surface at this point. These isophase lines describe the element (O) of the surface at this point. These isophase lines describe the element (O) of the surface of the wave aberration. The projection of such isophase lines from the point (σ_0) on the discharge pupil plane represents ~~an~~ an isophase line in the vicinity of the pupil point (O). We will designate by σ_0 the linear distances from the point (O) in the tangent plane in the cross-section, which corresponds to the examined cross-section of the pupil element (O)! The wave aberration, expressed in terms of σ_0 , is:

$$\left. l_\Omega(\sigma_0) \right|_{\sigma_0 \rightarrow 0} = \frac{\Delta}{2r^2} \sigma_0^2. \quad (3) \quad p307$$

From the general expression for the ~~curvature~~ radius we can see that the ~~curvature~~ radius R_Ω at the point (O) of the wave-aberration surface in the ~~cross-section considered~~ examined is equal to:

$$R_\Omega = \frac{[\Delta + (l_\Omega')^2]^{1/2}}{l_\Omega''} = \frac{r^2}{\Delta} \quad (4) \quad p307$$

Allowing for (3), we can rewrite formula (4) thus:

$$\frac{\sigma_0^2}{2l_\Omega(\sigma_0)} \Big|_{\sigma_0 \rightarrow 0} = R_\Omega. \quad (5)$$

A Dupin indicatrix can be constructed for the point (0) of the wave-aberration surface. To do this, in the tangential plane to the surface at the point (0) it is necessary to plot from the point (0), along each tangent segment, a distance equal to the square root of the absolute magnitude of the curvature radius corresponding to the normal cross-section of the wave-aberration surface element (0); i.e. the quantity $A = \sqrt{|R_{\alpha}|}$. Dupin's indicatrix is the central curve of the second order with center at the point (0). On the other hand, from formula (5) we get

$$\sigma_0 = \sqrt{2l_B(\sigma_0)} \cdot \sqrt{|R_{\alpha}|} \quad (6) \quad |0,307$$

If $l_B(\sigma_0) = \text{const.}$ is given, some kind of values for various normal cross-sections of the wave-aberration surface element (0), then the values of σ_0 , which is obtained from (6), plotted along the corresponding tangents to this element, form a geometrical locus of points of similar values of wave aberration in the vicinity of the point (0); i.e. an isophase line. From (6) it is clear that the isophase line in the near vicinity of the point (0) is a central curve of the second degree with center at point (0) similar to Dupin's indicatrix. It is obtained if all the radius vectors, plotted from point (0) in the tangent plane, which are in magnitude to $\sqrt{|R_{\alpha}|}$ (vertical lines) and whose ends form the Dupin indicatrix, changed $\sqrt{2l_B(\sigma_0)} = \text{const.}$ times.

Let us consider what occurs with the element (0) of the wave-aberration surface and the element of the isophase lines, corresponding to the wave-surface element (0), when the center (S) of the compared sphere tangent at the point (0) is moved along the normal to the mentioned wave-surface element (along the ray). Here it occupies successively different positions in (S) determining the dispersion-spot points in various planes of the arrangement connected with the general normal.

The wave surface can contain astigmatic and anastigmatic elements.

Let us examine the case of the wave-surface astigmatic element of a.

For any location of the point (Ω) on the ray, outside of its segment, containing curvature centers of normal cross-sections of the wave-surface element (O), the point (O) of the wave-aberration surface is elliptical. The isophase^{sic} line is an ellipse with center at point (O) and with axes which coincide with the directions of the main cross-sections of the wave-surface element.

At a position of the point (Ω) within the mentioned ray segment, the point (O) of the wave-aberration surface is a hyperbole. Isophase^{sic} line is represented here as a pair of coupled hyperbolae with center at the point (O) and with axes oriented as in the previous case. The asymptotes of these hyperbolae represent a pair of normal cross-sections of the wave surface^{of the} element (O), whose curvature centers coincide with the point (Ω). The angle W_0 between the asymptotes is described:

by the expression

$$\Delta_u, \Delta_v: \quad \operatorname{tg}^2 \frac{W_0}{2} = - \frac{\Delta_u}{\Delta_v} \quad (7)$$

where the Δ 's are the distances along the ray from the point (Ω) to the curvature centers of the main cross-sections of the wave-surface element (O). From (7) it is clear that, with a shift of the studied point of the dispersion spot along the ray within the limits of the segment of the curvature centers, the angle between the asymptotes decreases. The angle between them becomes zero when the point (Ω) coincides with the curvature center of each of the pairs of the main cross-sections of the wave-surface elements. Here the hyperbolae^{sic} degenerate into straight lines having a direction which corresponds to the main cross-section. The hyperbole asymptotes in the discharge pupil plane are represented by the above-mentioned projections from the point (Ω) of the asymptotes in the plane which is tangent to the wave-surface element (O).

The point (O) of the wave-aberration surface is a parabola when

the point (σ_2) coincides with the curvature center of the main cross-section of the wave-surface element (0). Simultaneously in this cross-section $l_{\sigma_2}'|_{\sigma_2=0} = 0$ and $l_{\sigma_2}''|_{\sigma_2=0} = 0$; in the remaining main cross-section only $l_{\sigma_2}'|_{\sigma_2=0} = 0$.

On the last cross-section there is accordingly a maximum or minimum correspondingly of the position of the point (σ_2) relative to the curvature center of the remaining main cross-section. The isophase line is represented here by a pair of straight lines, parallel to the main cross-section, whose curvature center coincides with the point (σ_2).

In the case of the anastigmatic wave-surface element (0), the wave-aberration surface element (0) remains an element of the rotation paraboloid for all positions of the point (σ_2) on the ray. When the point (σ_2) coincides with the curvature center of the wave-surface

element (0), then we have simultaneously: $l_{\sigma_2}'|_{\sigma_2=0} = 0$ and $l_{\sigma_2}''|_{\sigma_2=0} = 0$ (7)

in all normal cross-sections of this element. When the point (σ_2) does not coincide with the mentioned curvature center, then we have $l_{\sigma_2}''|_{\sigma_2=0} = 0$ and $l_{\sigma_2}''|_{\sigma_2=0} \leq 0$ i.e., it is a maximum or minimum depending on the position of the point (σ_2) on the ray relative to the curvature center.

The isophase line is a circle for all positions of the point (σ_2).

In this case, the wave-aberration surface point is umbilical.

If the point (σ_2) coincides with the curvature center, then the point (0) of the wave-aberration surface is focal umbilical (or simply umbilical). In the opposite case it is not focal umbilical.

Thus, when the movement of the studied point (σ_2) along the normal to the astigmatic wave-surface element (0), the point (0) of the wave-aberration surface becomes in turn elliptical -- parabolic -- hyperbola -- parabolic -- elliptical. At the same time, the appearance of the isophase line varies near a circle changes in the vicinity of the pupil element (0) as well does the value of l_{σ_2}'' at the point (0) in various cross-sections of the pupil element (0) which corresponds to various normal cross-sections of

wave- *for*
 the surface element (0) [in all positions] of the point ($\beta\theta$) on the ray we still have $\ell_{\alpha}^{\prime \prime} = 0$ at the point (0). In the case of the astigmatic wave-surface element (0), the wave-aberration surface point (0) gradually successively becomes nonfocal umbilical -- umbilical -- nonfocal umbilical.

Intersection of the *junction* of the normals to the wave surface with any focusing plane makes the point ($\beta\theta$) an energy-dispersion spot.

Each point ($\beta\theta$) of the energy-dispersion spot can be regarded as *center* of a comparison sphere tangent to the wave surface at the point (0), the normal of which contains this point and can be constructed for the point ($\beta\theta$) *the wave surface aberration*. Elements (0) of the wave-aberration surface corresponding to various points ($\beta\theta$) of the dispersion spot in the given focusing spot, as centers of comparison spheres which are tangent (spheres) to the wave surface at the point (0), can be of five types: elliptical, hyperbolae, parabolas, umbilical, and nonfocal umbilical.

start here
 The properties were examined above of the isophase line in the plane tangent to the wave surface at the point (0), from which the normal contains the examined point ($\beta\theta$) of the dispersion spot. This normal coincides with the ray *arriving* from the point (0) of the discharge pupil system. The isophase line in the vicinity of the pupil point (0) is represented as the projection from the point ($\beta\theta$) of the isophase line in the plane which is tangent to the wave surface at point (0).

We can agree to ~~each~~ name each point (0) of the discharge pupil system *according to* the type to which the wave-aberration surface point (0) belongs, constructed for the studied point ($\beta\theta$) of the energy-dispersion spot belonging to the ray from the pupil point (0). Thus for given positions of the luminous point and focusing planes, the discharge pupil system points can be classified. They can be of five types: elliptical, hyperbolae, parabolas, umbilical, and nonfocal umbilical. Depending upon the wave-surface structure and focusing planes, the pupils can contain all of the types or some of them may be absent.

The ray from the umbilical 'pupil' point ~~arrived at~~ the focal nucleus. The ray from the parabolic 'pupil' point ~~arrived at~~ the picture element of the focal lines. The ray from the elliptical and nonfocal umbilical ~~pupil points arrived at~~ the outer diffusion element of the dispersion spot. The ray from the hyperbolic 'pupil' point ~~arrived at~~ occurred at the inner diffusion element of the dispersion spot.

To various normal cross-sections of the wave-surface element (0), there correspond cross-sections of the 'pupil' element (0) formed by these intersecting planes. We will call the cross-section of the 'pupil' element (0), corresponding to the main cross-sections of the wave-surface element (0), the main cross-section of the 'pupil' element (0).

The comparison sphere, constructed around a certain point (Ω) of the dispersion spot, as around a center, can appear tangent to the wave surface ~~in many places~~ ^{at several points} ~~at the same time or within the limits of~~ ⁱⁿ a whole region of its points. The normals to the wave surface of these points make the 'pupil' points uniform or of various types. The light disturbance from these 'pupil' points ~~occurred~~ ^{arose} ~~in~~ ^{the same} the point (Ω) in a uniform phase. At the point (Ω), ~~there occurs the need~~ ^{the same} of the dispersion-spot elements of ~~a uniform~~ ^{the same} or different types. A change of the comparison sphere, which is tangent at a certain point (0) of the wave surface, to one concentric with it, does not change the type to which the pupil point belongs. The comparison sphere, tangent at a certain point (0) of the wave surface, can appear concentric with the comparison sphere tangent at another point of the wave surface. The phases, with which the light disturbance ^(arose at) ~~occurred~~ ^{from the corresponding pupil points}, ~~can appear as various~~ different.

If the dispersion-spot point (Ω) does not belong to any normal to the wave surface, then the comparison sphere constructed around it does not ~~occur~~ ^{lie} tangent to the wave surface at any point, and the wave aberration surface constructed for it does not have any "extreme" sections. The dispersion-spot element around such a point (Ω) is ~~diffusionary~~ ^{extremal parts} diffractionary, diffusive-diffractive.

2. A Method of Experimentally Studying the Connection Between the ~~of the~~
Wave-Surface Structure and the Structure of the Energy-Dispersion
Spots.

The proposed method (method of "eye" interferograms), which is easily accomplished on Twyman's (1) or Linnik's (2) interferometer, allows one to observe the ~~structure~~ of the examined object ^{under study} in the discharge pupil plane, or in any other plane, the interferograms for various points (Ω), of the energy-dispersion spot, which coincide with the center of the comparison sphere. Such an interferogram represents the above-examined family of isoberration (isophase) lines which describe the wave-aberration surface for the given point (Ω). The "eyes" of the interferograms will be called the elements which correspond to the extreme parts of the wave-aberration surface for the point (Ω). The "eyes" specify the pupil elements from which the rays arrive to the point (Ω). The interference band in the vicinity of the pupil point (0), from which the ray comes to the point (Ω) which coincides with the center of the comparison sphere, represents the projection from the point (Ω) of Dupin's indicatrix which is constructed for the wave-surface point (0). The "eyes" possess characteristic properties depending upon the type to which the discharge pupil, surrounded by them belongs. This permits one an experimentally classification of the discharge-pupil point of the examined system for given positions of the emitter and focusing plane. One can immediately observe various types of 'pupil' elements, and the properties of their formation, and their coordinates x, y are determined. In particular, the region area ^{near} ~~of~~ observe which is found surrounding the pupil point (0) of a given type (from which the ray arrives at point (Ω)) and sending into the point (Ω) light disturbances of various phase which do not exceed a certain given amount (for example $\pi/2$). Simultaneously, one obtains the coordinates ξ, η of the point (Ω) experimentally obtained for which the interferogram and its "eyes" are observed. The type to which 'pupil' point (0) sending a ray to the point (Ω) belongs determines the type to which

the energy-dispersion spot element [of the near] point (Ω_2) belongs.

Observation
 From the immediate findings of the interference pictures on the discharge 'pupil' of the examined optical system, obtained for various points (Ω_2) of the energy-dispersion spot, this spot can be constructed; that is, like its focal nucleus, the picture of the focal lines and diffusion elements can be constructed and evaluated their brightness made. At the same time, the geometric place of the 'pupil' points can be constructed from which the rays pass into the points of the energy-dispersion spot of a given type. (Immediately) The role of various 'pupil' segments can be evaluated in forming the description.

By a comparison of energy-dispersion spots, constructed with the aid of the interferometer, with the findings in actually similar arrangement, the properties of various types of energy-dispersion spot elements of with the properties of corresponding type of discharge-pupil elements of optical systems were connected.

Thus, the proposed method of interferogram "eyes" ensures the possibility of an immediate evaluation of the role of various segments parts of the discharge pupil system in forming the description of the given element of its pupil field, and descriptive properties of the image.

The center of the comparison sphere, combined with various points (Ω_2) of the energy-dispersion spot, in Turymann's interferometer is represented as the center of specific spectrum of the instrument. In the case of Linnik's interferometer it is represented by an opening which appears as the source of spherical diffraction waves. Joining the center of the compared sphere with various points (Ω_2) of the energy-dispersion spot, each time can be recorded when the coordinates ξ, η of the point (Ω_2) are found for the interferogram. On the other hand, the coordinates of the 'pupil' point can be registered by the interferogram from which rays go to point (Ω_2).

(Twyman's) In Twyman's interferometer, instead of joining the center of a spherical mirror ^{shifting} in ^{fixed} its position in a certain focusing plane, by maintaining its position the angle of inclination can be changed between the wave-comparison plane and the waves arising from the studied object surface (surface of wave aberration). The coordinates ξ_1, η_1 corresponding to the point (Ω_1) of the energy-dispersion spot, are easily obtained from the components of the waves making up this inclination angle. The obtained interferogram also appears as if the center of the specific spectrum had been shifted to be placed at the point (Ω_2) with these coordinates ξ_2, η_2 . Actually, from the wave-aberration surface $l_{\Omega_2} = l_{\Omega_1}(x, y)$ for any point (Ω_2) of the dispersion spot, Cartesian coordinates which are ξ_2, η_2 , can be transformed to wave-aberration surface $l_{\Omega_2} = l_{\Omega_1}(x, y)$ for any other of its points with coordinates ξ_2, η_2 . To this corresponds the shifting joining of the centers of the compared sphere in an unchangeable focusing plane, equal (shifting) to $S\xi = \xi_2 - \xi_1, S\eta = \eta_2 - \eta_1$.

If r is the distance from the discharge pupil system to the focusing plane and $S\xi, S\eta \ll r$, then the indicated transfer occurs with the aid of the expression $l_{\Omega_2} = l_{\Omega_1} + Ax + By + C$

where A, B, C are constants; $A = \frac{S\xi}{r}, B = \frac{S\eta}{r}$ and C can be made equal to any constant. In other words, to the values of the wave aberration $l_{\Omega_1}(x, y)$, deviations with their sign are added from the x, y plane inclined to the dimension which is determined by the values of the coefficients A, B .

Thus, if, for a constant ^{of} in an unchanging position (Ω_1) of the center C of the spherical mirror whose coordinates are ξ_1, η_1 , the wave comparison plane is turned relative to the waves returning from the studied object (or vice versa) by a certain angle u , then the obtained interferogram will be such as if the center of the specific spectrum had been displaced from the point (Ω_1) with coordinates ξ_1, η_1 , by the amount :

$$\begin{aligned} S\xi &= Ar = u_x r \\ S\eta &= Br = u_y r \end{aligned}$$

are the components of σ (extended)
 where u_x , u_y make up the angle of turn, and is led to the point (Ω_2)
 with the coordinates $\xi_2 = \xi_1 + u_x r$ and $\eta_2 = \eta_1 + u_y r$.

Interference bands (isoberration lines) in the vicinity of the 'pupil' points (0) from which the rays ~~come to~~^{leave at} the given point (Ω_2) of the dispersion spot, combined with the center of the comparison sphere, have a characteristic form ("eyes" of the interferogram), depending on the type to which the 'pupil' point (0) belongs for the examined focusing plane and given position of the point emitter. Since the mentioned 'pupil' points (0) correspond to the experimental points (0) of the wave-aberration surface for the point (Ω_2), then the elements (0) of the interference picture have another characteristic property. They appear as "centers of disturbances" of the interference picture. This signifies that if the difference ~~is continuously varied in~~^{is simultaneously varied in} the optical paths between interference waves (for example, pressure of the finger on the interferometer) and at the same time take the interference picture, then it can be noticed that the bands of the picture are ~~combined~~^{shifted}, drawn to toward the indicated centers of disturbance or ~~separating~~^{away} from them. The "centers of disturbance remain ~~immobile~~^{immobile} on the picture. During this,

The following corresponds to an increase of the difference of the optical paths between the interference waves. The bands diverge from the center of disturbance ~~at~~^{at} the point (0) of the picture, if the wave-aberration surface point (0) is a maximum in the examined cross-section. The bands are drawn to the center of disturbance in the case of a minimum. If the point (0) of the wave-aberration surface is the ~~curvature~~^a point of ~~in~~^{on} the examined cross-section, then the bands from one side are drawn toward to the center of disturbance and from the other are drawn way from it.

Let rays pass through the point (Ω_2) coming from several 'pupil' points. Correspondingly there occur several centers of disturbance of the interferogram. During continuous variation in the examined cross-section, the dispersion spot which determines the position of the points (Ω_2) of the dispersion spot, the position of the points of its tangency

(In 1924, Perry made use of the phenomenon of "disturbance centers" in interferograms in order to measure the geometric aberration of objectives (3)).

to the wave surface arising from the studied object (wave-aberration surface) varies continuously. The wave comparison plane can be inclined so that the adjacent centers of disturbance of the interferogram are displaced toward each other. At ~~some~~^{45°} the value of wave aberration between them becomes all the smaller and, at a certain inclination of the wave comparison plane, it becomes equal to zero; both "disturbance zones" run together forming an extension of each other. When "centers of disturbance" are drawn close to each other ~~and having~~^{representing} elliptical, hyperbolic, nonfocal umbilical 'pupil' points, then a parabolic 'pupil' point is formed in the corresponding cross-section. When "centers of disturbance" are brought together, representing parabolic 'pupil' points in their cross-sections that are orthogonal to the direction of approach, then an umbilical 'pupil' point is formed. If the "centers of disturbance" approach each other representing parabolic 'pupil' points in their cross-sections coinciding with the direction of approach, then an parabolic area of 'pupil' points is formed. In the formation of a parabolic 'pupil' point, the line joining the approaching "centers of disturbance" determines the main cross-section of the 'pupil' element which corresponds to the direction of the main cross-section of the wave surface element.

Descriptions of the properties of elements in an interferogram around 'pupil' points (O), from which rays go to point (Ω) , combined with the center of the comparison sphere, permits the localization of these pupil points (O) and determination of their type.

Thus with the aid of the interferometer, the energy-dispersion spot (its focal nucleus, focal lines and diffusion elements) can be immediately constructed at any focusing point or any section of the luminous point! The geometric place of the 'pupil' points of a given type can be constructed. In particular, the geometric place of the 'pupil' points can be constructed from which rays arrive at the focal elements of the dispersion spot. The caustic surface can be constructed for a given position of the luminous point.

The above-indicated constructions are immediately obtained from observations of the apparatus. The amount of turn of the wave comparison findings on the apparatus. The size of ~~the~~ turn of the wave comparison gives [plane] relative to the wave surface arising from the studied object gives the coordinates (ξ, η) of the dispersion-spot point (P). With the aid of an ocular micrometer provided with an additional lens, the coordinates of the 'pupil' points from which rays go to the point (P) are determined for a given angle of turn of the wave comparison plane. Using the above-described criteria, the type ~~is~~ determined to which these pupil points belong and, correspondingly, the type to which the dispersion-spot element around the point (P) belongs. Therefore the intersection of the caustic surface with the focusing plane for a given position of the luminous point can also be constructed based on the results of the work on a photograph of an interferogram, as was done by Shyusamer [4]. To do this it is necessary to use the very laborious half graphic, half calculation method. The above-described method permits a solution of this problem from the immediate findings on the interferometer noticeably faster and conveniently. Actually the matter here leads to certain manipulations and readings along the scale. Moreover the proposed method permits the immediate evaluation of the comparative brightness of the elements of the picture of the focal lines, the finding of the properties of the 'pupil' elements around the points of the various types and determination of their role in the formation of the detailed description.

Fig. 1 shows an interferogram photograph, obtained by the above-described method. Examples of discharged pupil points of various types are included. The photographs relate to Tessar ^(were made with a) Zeiss photo-objective $f = 300, 1:4.5$ and were obtained in the light of a sodium lamp for various angular distances of the luminous point from the optical axis of the object and for various focusing planes. The white spot on the interferogram shows the position of the pupil center (beginning of the coordinate system). The dark points on the interferogram indicate the position of the various focusing planes. The white spot on the interferogram indicates the position of the pupil center (beginning of the coordinate system). The dark points on the interferogram show the location of the various focusing planes.

(under examination)

of the examined pupil points. d will designate the distances from the focusing plane, containing the (the sagittal) focus of "umbilical" cross-section of the pupil element around its center, to the examined focusing plane, read off along the main ray,

Photograph 1. $\Phi = 20^\circ$; $d_s = 0.0$; $\xi = -0.054 \text{ mm}$; $\eta = 0$.

(under study.)

The 'pupil' point is umbilical. The point (Ω) of the dispersion spot with the indicated coordinates, to which the ray comes from the examined 'pupil' point, belongs to the focal nucleus.

Photo 2. $\Phi = 10^\circ$; $d_s = 0.2 \text{ mm}$; $\xi = -0.01 \text{ mm}$; $\eta = -0.03 \text{ mm}$

The 'pupil' point is umbilical. The point (Ω) of the dispersion spot belongs to the focal nucleus. To this point (Ω), a ray comes from the elliptical 'pupil' point (below); the diffusion element of the dispersion spot is superimposed on the focal nucleus.

Photo 3. $\Phi = 0^\circ$; $d_s = -1.1 \text{ mm}$; $\xi = 0$; $\eta = 0$.

Here there occurs an area of umbilical points distributed around the circle, from which rays pass to the point (Ω) of the dispersion spot with the indicated coordinates. The point (Ω) belongs to the focal nucleus. To this point (Ω) there passes a ray from the 'pupil' center, with which its nonfocal umbilical point coincides.

Photo 4. $\Phi = 0^\circ$; $d_s = 0.0$; $\xi = 0$; $\eta = 0$.

In the 'pupil' center and around it is an area of umbilical points. Rays from them pass the point (Ω) which belongs to the focal nucleus.

The point (Ω) coincides with the focus of the paraxial rays.

Photo 5. $\Phi = 15^\circ$; $d_s = 0.0$; $\xi = 0.0$; $\eta = 0.0$

There occurs an area of parabolic 'pupil' points and three of its elliptical points.

Photo 6. $\Phi = 23^\circ$; $d_s = 0.0$; $\xi = -0.015 \text{ mm}$; $\eta = 0$

There occurs an area of parabolic points and one elliptical.

Photo 7. $\Phi = 0^\circ$; $d_s = -0.9 \text{ mm}$; $\xi = 0$; $\eta = 0$.

Here there exist two areas of parabolic points, distributed in

~~close to~~
 concentric circles from which rays pass approximately into one point (Ω). The point (Ω) belongs to an astigmatic focal nucleus. In the center ~~there~~ is found a non-focal umbilical 'pupil' point.

Foto 8. $\Phi = 12.5^\circ$; $d_s = +0.9 \text{ mm}$; $\xi = -0.015 \text{ mm}$; $\eta = -0.036 \text{ mm}$.

There occurs ^{region} an area of parabolic 'pupil' points from which rays arrive at ~~pass~~ the point (Ω) with the indicated coordinates.

Foto 9. $\Phi = 25^\circ$; $d_s = +2.1 \text{ mm}$; $\xi = 0$; $\eta = -0.09 \text{ mm}$.

The 'pupil' point is parabolic. The ray from it ~~passes~~ arrives at the focal element of the dispersion spot, located around the point with the indicated coordinates. The direction of the main cross-section is shown by the dotted line.

Foto 10. $\Phi = 0^\circ$; $d_s = -0.9 \text{ mm}$; $\xi = -0.017 \text{ mm}$; $\eta = 0$.

In the point (ω) of the dispersion spot with these coordinates pass the rays from two parabolic 'pupil' points and one elliptical. In the point (Ω) elements meet of two focal lines belonging to two bands of the caustic surface at \downarrow .

Foto 11. $\Phi = 25^\circ$; $d_s = +0.6 \text{ mm}$; $\xi = +0.045 \text{ mm}$; $\eta = +0.005 \text{ mm}$.

In the point (Ω) with these coordinates, rays pass from two elliptical 'pupil' points, from two of its hyperbola points and from one parabolic.

Foto 12. $\Phi = 23^\circ$; $d_s = +1.7 \text{ mm}$; $\xi = +0.025 \text{ mm}$; $\eta = 0$.

In the point (ω) with these points, rays pass from four 'pupil' points - three elliptical and one hyperbolic.

The dispersion spots, constructed with the aid of the interferometer, were compared with findings in actual conditions for the same length of light ^(those observed under) wave, ^{positions} location of the luminous point and focusing plane. The purpose of the comparison was to study the properties of the dispersion-spot elements of various types in connection with the properties of 'pupil' elements from which rays pass into these dispersion-spot elements. The findings and photography were done with the aid of an ^{observations effect}

apparatus

Instrument with a 'nodal carriage' placed before the collimator. In the focus of the collimator ~~four circular openings were placed~~ of various diameter which were illuminated by a sodium lamp. The mechanism of the instrument with the nodal carriage was the same as Turym's Twyman's interferometer. In place of the center of the spherical mirror was the object point of the microscope. In place of the microscope's eyepiece there could be place an ocular-micrometer or ~~speculum~~ phot-apparatus with a ~~leveled~~ objective. In both instruments the ~~studied~~ ^{under} object is so placed that its second nodal point is ~~fixed~~ on the axis of (its) rotation. This is ensured the similarity of the focusing planes and position of the 'luminous point' on both instruments. Micro-objectives apertures were used with ^{an} exceeding apertures which surpass the aperture of the a high-quality ~~studied~~ objective.

The schemes and photographs of Fig. 2 illustrate the use of the above-described method of working with Turym's (Twyman's) interferometer and compared the dispersion spot constructed with the interferometer with observations actual findings. The data given in Fig. 2, relate to the Tessar Zeiss Teeyes photo-objective $f = 300$, 1:4.5 at an angular distance of the luminous point from the optical axis of $\Phi = 15^\circ$. The distance from the second 'nodal' point of the object to the focusing plane is 211.4 mm.

Interferograms describe the position of the parabolic 'pupil' points. Under each interferogram there are indicated, in microns, the coordinates ξ, η of the point (S_2) of the dispersion spot belonging to the focal line element, to which the ray passes from the examined parabolic pupil ^{under dimension}. In scheme 1 there are constructed the discharge pupils of the object and on it the geometric place of the parabolic 'pupil' points. Here the coordinates ξ, η are also indicated (expressed in microns) of the dispersion-spot points, to which the rays pass from these 'pupil' points. In scheme 2, the picture is shown of the dispersion-spot focal lines, which were constructed with the aid of the interferometer ^(picture) according to the above-described method. Around the points

of this picture), the coordinates x , y [are shown] (expressed in millimetres) of the pupil points, from which rays pass [to] these points.

An enlargement is [not] shown here of the photograph of dispersion spot obtained [photograph] under conditions obtained by the apparatus with the nodal carriage arrangement similar to those for which it was constructed by the interferometer.

~~These~~ arrangements, similar to these for Fig 2, were ~~set up~~ constructed for various angular distances of the luminous point from the object axis and for various focusing planes.

Later it appeared possible, by way of certain readjustments of the interferometer, to ~~find~~ ^{observe} and to immediately photograph on the interferometer an enlarged energy-dispersion spot by placing ~~stems~~ ^{the of dots} ("probes") on it, making a dispersion-spot element ~~to~~ ^{from} which rays from the "eyes" of the interferogram pass. The latter are ~~found~~ ^{observed} and photographed simultaneously. Thus, the ~~connection~~ ^{relation} between the pupil elements and the energy-dispersion spot elements is graphically represented on photographs which can be easily measured and ~~analyzed~~ ^{therefore} ~~analyzed~~.

There occurs a good correspondence ^{exists between} of the energy-dispersion spots, constructed with the aid of the interferometer, and actual observations.

Submitted for Publication

22 Nov. 1947

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T. N. Fig 1 - Photographs

Fig 2 - Photographs and 2 complicated graphs.

See page 316
of ZTF
Vol XVIII
No 3 (Mar '48)